Field Theory Analysis of Circular Ridge Waveguides with Partial Dielectric Filling

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Abstract: The circular ridge waveguide (CRW) is a very useful structure for tuningless dual mode filters and septum polarizers. In this paper we present a rigorous analysis of the mode spectrum of CRW which is based on the cylindrical method of lines (CMoL). The advantage of the CMoL is that only one space variable need to be discretized and that spurious modes as well as relative convergence phenomena are not encountered. Partial dielectric filling as well as structures with mixed cylindrical/ rectangular boundaries can be easily included in the analysis. Results will be given for homogeneously filled CRW and for rectangular waveguides with cylindrical dielectric blocks.

I. Introduction

Circular ridge waveguides (CRW) are useful for the design of tuningless dual mode filters and septum polarizers [1].Successful design of these components with CRW depends largely on the accurate characterization of the CRW section. So far only one attempt has been published to calculate the mode spectrum of asymmetric CRW using the finite element method (FEM) [1].In that paper the ridges were shaped to fit the cylindrical coordinate system (Fig.1b). If a cylindrical discretization scheme is employed, this measure is useful to avoid discretization of rectangular structural details. The idea of shaping the structure to avoid rectangular ridges in a cylindrical coordinate system has been successfully applied before to the mode matching analysis of metal septum loaded circular waveguide filters [2]. With this step mathematical complications in the solution of the coupling integrals could be eliminated. The good agreement between measured and predicted results ([2]) has justified this step. Furthermore, in particular when the ridge thickness is such that milling techniques can be applied, it should make no difference whether or not the ridges are of rectangular of angled shape.

Accurate characterization of the CRW structure requires a large number of higher order modes. In particular if the CRW is to be used as a section in a dual mode resonator. To represent also the highest order mode with sufficient accuracy, the finite element analysis involves a very fine two-dimensional discretization.This leads not only to large computer memory and long computation time, but introduces also uncertainties with respect to the solutions, because of the potential for spurious modes in the FEM. To avoid these problems we have utilized the cylindrical method of lines (CMoL) [3] instead. The CMoL is a finite difference method, which discretizes only the angular space direction of the CRW while analytical solutions are sought for the radial direction. In comparison to finite element or finite difference methods, which require a two-dimensional discretization, the CMoL consumes much less computer memory space and leads also to a much simpler discretization scheme. Since spurious solutions and relative convergence problems are not encountered, the method shows great potential for fast and accurate CAD of waveguide components. The following outlines the principle steps of the CMoL. By taking advantage of the singular value decomposition technique [4], poles in the determinant calculation are eliminated. Results are presented for the mode spectrum of CRW sections with four and five ridges, respectively. The flexibility of the method is illustrated by calculating

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also the mode spectrum of circular dielectric rods in a rectangular waveguide housing and vice versa.

Theory

The generalized waveguide cross-section for which the method is derived is shown in Fig.1a. The electromagnetic fields in the uniform region can be derived from the longitudinal field components $\phi_{e,h} \propto (E_z, H_z)$. Both field components satisfy the source-free Helmholtz equation in polar coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right) + \frac{\partial^2\phi}{r^2\partial\theta^2} + (k^2 - \beta^2)\phi = 0 \qquad (1)$$

and the boundary conditions depending on the structure and modes we are interested in, where $k^2 = \omega^2\mu\varepsilon$. Since the problem can not be solved analytically for the whole region, the calculation domain is discretized along the angular direction by N straight lines in r-direction. This yields for the θ-variables

$$\phi_k = \phi_1 + (k-1)h = \frac{2\pi k}{N}, \text{k=1,2,...,N} \qquad (2)$$

with $h = 2\pi/N$ being the angular spacing between them. Using the central finite difference scheme for the angular variable θ, a second order finite difference operator [P] can be found which depends on the lateral boundary conditions. For generally asymmetric structures, [P] can be found in [3].

The discretized Helmholtz equation is now represented by a set of coupled ordinary differential equations

$$\frac{d}{rdr}\left(r\frac{d\bar{\phi}}{dr}\right) + \lambda^2\bar{\phi} - \frac{[P]\bar{\phi}}{r^2h^2} = \delta, \delta = o(h^2) \qquad (3)$$

where $\lambda^2 = k^2 - \beta^2$ for inhomogeneous waveguides; $\lambda^2 = 0$ for TEM transmission lines or static problems; $\lambda^2 = k^2_c$ for TE and TM analysis of homogeneous waveguides. Multiplying equ. (3) with an orthogonal transformation matrix [T] [3]

$$T_{ki} = \{\cos\alpha_{ki} + \sin\alpha_{ki}\}/\sqrt{N},$$

$$\alpha_{ki} = ikh, h=2\pi/N, i,k=1,2,...,N$$

from the left and right sides, respectively, matrix [P] can be diagonalized. This yields a set of decoupled Helmholtz equations of the following form

$$\frac{d}{rdr}\left(r\frac{du_i}{dr}\right) + \left(\lambda^2 - \frac{\lambda_i^2}{r^2}\right)u_i = 0 \qquad (4)$$

$$\lambda_i = \frac{2\sin(\varphi_i/2)}{h} \qquad (5)$$

where u_i (i = 1,2,3,...,N) is called the transformed potential function and $\bar{u} = [u_1, u_2, ..., u_N] = [T]\bar{\phi}$.

In every uniform region, a solution of equ. (4) may be written as a superposition of Bessel functions. The selection of different types of Bessel functions depends on the individual applications. For the guided-wave structures considered here, a combination of Bessel functions of λ_i-order can be used

$$u_i = A_i J_{\lambda_i}(\lambda r) + B_i N_{\lambda_i}(\lambda r) \qquad (6)$$

TE/TM Cutoff Frequencies of Homogeneous Waveguides

Taking a homogeneous waveguide with arbitrary contour an example ($\varepsilon_1 = \varepsilon_2$ in Fig. 1a), the finite field values at r = 0 require $B_i = 0$. This leads directly to the characteristic equation for TM modes

$$[T]\,[A_k J_{\lambda_i}(k_c R_k)] = \left[T_{ki} J_{\lambda_i}(k_c R_k)\right]\bar{A} = 0 \qquad (7)$$

where R_k is the radius of the k-th discretization line. In other words, one needs only to input the radius R_k of the contour (k=1,2,3,...,N) at each discretization line and then solve for

$$det\left(\left[T_{ki} J_{\lambda_i}(k_c R_k)\right]\right) = 0 \qquad (8)$$

where $J_{\lambda i}(k_c R_k)$ is the λ_i-order Bessel function of $k_c R_k$. Similarly, TE modes can be obtained if we substitute $J_{\lambda i}(k_c R_k)$ in (8) with

$$J_{\lambda_i}(k_c R_k) = \cos(\bar{r}_k, \bar{n}_k)\frac{dJ_{\lambda_i}(k_c R_k)}{dr} \qquad (9)$$

where, $\bar{r}_k$ and $\bar{n}_k$ are, respectively, the unit vectors along the k-th radial line and the outwards normal vector to the contour.

For the ridged circular waveguide (Fig.1b), the cross-section is subdivided into several homogeneous subregions. The number of these subregions is determined by the number of ridges. In each subregion, the solution to equ. (4) is given by equ. (5). Since the subregion may contain different material, the field continuity conditions at interfaces between region I and the other subregions must be included. Since all the electromagnetic fields can be related to the potential functions

$\phi_{e,h} \propto (E_z, H_z)$ as

$$\bar{E} = \frac{1}{j\omega\varepsilon}\nabla\times\nabla(\phi_e\hat{z}) - \nabla(\phi_h\hat{z}) \quad (10)$$

$$\bar{H} = \frac{1}{j\omega\mu}\nabla\times\nabla(\phi_h\hat{z}) + \nabla(\phi_e\hat{z}) \quad (11)$$

The field continuity condition, for example, for the E_θ yields

$$\frac{\beta}{\omega\varepsilon_0}[D]\left(\frac{\bar{\phi}_e^I}{\varepsilon_{r,I}} - \frac{\bar{\phi}_e^{II}}{\varepsilon_{r,II}}\right) = rh\left(\frac{d\bar{\phi}_h^{II}}{dr} - \frac{d\bar{\phi}_h^I}{dr}\right) \quad (12)$$

[D] denotes the bi-diagonal first order finite difference operator. Multiplying $[T]^t$ and [T] from the left and right sides, respectively, the above equation is diagonalized and yields one equation per line for the potential u ($\bar{u}=[T]\bar{\phi}$).

The remaining three steps are: 1. transform the known fields from the boundary of the system into the interface plane, which can be done with the following equation:

$$\begin{bmatrix} u_i(\lambda r_1) \\ \frac{1}{\lambda}\frac{du_i(\lambda r_1)}{dr} \end{bmatrix} = [w(J_{\lambda_i}(\lambda r_1), N_{\lambda_i}(\lambda r_1))] \times$$

$$[w(J_{\lambda_i}(\lambda r_2), N_{\lambda_i}(\lambda r_2))]^{-1} \times \begin{bmatrix} u_i(\lambda r_2) \\ \frac{1}{\lambda}\frac{du_i(\lambda r_2)}{dr} \end{bmatrix} \quad (13)$$

$$[w(y_1, y_2)] = \begin{bmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{bmatrix} \quad (14)$$

2. establish a relationship in the transformed domain between the tangential electric field and the surface current in the interface. 3. transform all transformed potential functions back to into the original domain to yield

$$[Z]\begin{bmatrix} E_z \\ E_\theta \end{bmatrix} = \begin{bmatrix} J_z \\ J_\theta \end{bmatrix} \quad (15)$$

Using the condition of zero current distribution in the interface between two regions, the zeros of

$$det\{[Z]\} = 0$$

must be found.

Numerical Results

To test the convergence of the orthogonal transformation matrix [T] obtained for the cylindrical method of lines, rectangular and circular waveguides are calculated and compared with analytical solutions. Convergence tests are shown in Fig.2 and Fig.3. It is interesting to note that convergence for the TM01 mode in Fig. 2 is possible with one line only, while the TE11 mode requires at last 15-20 lines. This can be explained by the fact that the field distribution of the TM01 mode is constant in angular direction and changes only in radial direction, which can be described by one line. The field of the TE11 mode, however, changes in radial and angular direction and thus requires more lines to be characterized accurately. Fig. 4 illustrates the effect of the fifth ridge, which is the coupling element in a dual mode CRW resonator. Here the influence on the first two TE modes with orthogonal polarization is small while the TM mode increases its cutoff frequency when the fifth ridge penetrates further into the CRW.

To demonstrate also the flexibility of the method when structures with mixed coordinate systems are combined, Fig. 5 shows the propagation constant of the fundamental hybrid mode in a cylindrical dielectric rod loaded rectangular waveguide. For the case of Fig. 5 50 lines were necessary to reach convergence. The typical computation time for the structures investigated was less than 2 minutes per frequency on a SUN SPARC 4 workstation. Since the computer code was not optimized this run time can be further improved by a factor of ten.

Conclusion

We have presented a rigorous calculation of the mode spectrum in circular ridge waveguides with five ridges and dielectric loaded rectangular waveguides containing circular dielectric rods. The cylindrical method of lines has been utilized and further developed to include also dielectric subsections.

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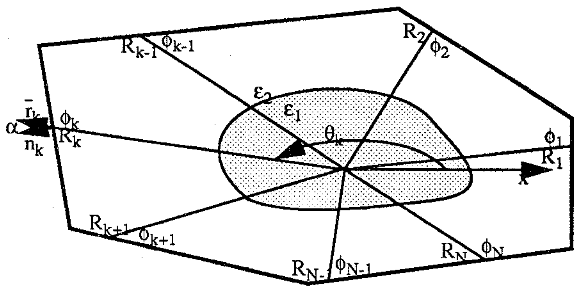

Fig. 1a MoL discretization in a cylindrical coordinate system

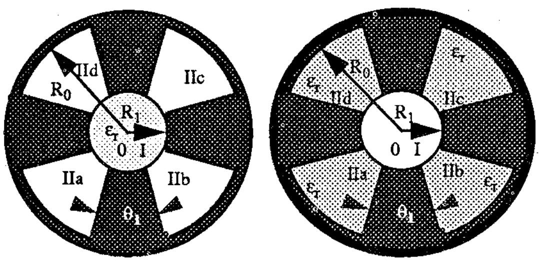

Fig. 1b Partially dielectric-loaded and circular ridged waveguide

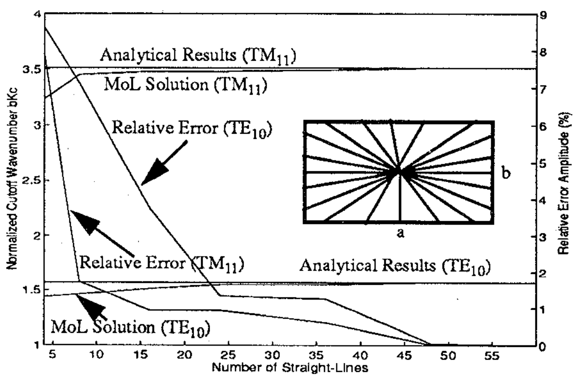

Fig. 2 MoL results for TE_{10} and TM_{11} modes compared to analytical solutions for a rectangular waveguide, b = a/2 = 3.555 mm (WR-28)

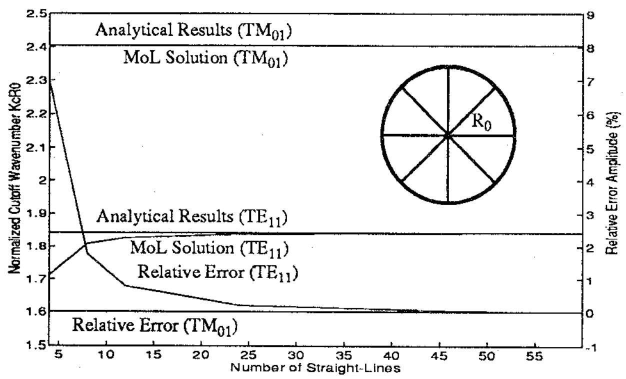

Fig. 3 MoL results for TE_{11} and TM_{01} modes compared to analytical solutions for a circular waveguide, R_0 = 2.54 cm

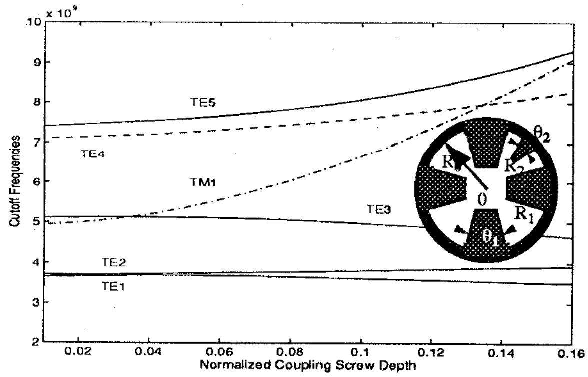

Fig.4 Cutoff frequencies of CRW with fifth ridge, R_0 = 2.54 cm, $\theta_1 = 25^0$, $\theta_2 = 12.5^0$

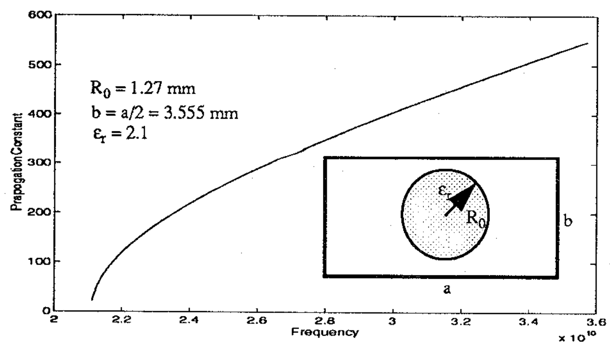

Fig. 5 Dispersion relation of fundamental mode in a dielectric-loaded rectangular waveguide, b = a/2 = 3.555 mm (WR-28), R_0 = 1.27 mm, ε_r = 2.1